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Dislocation models for fault creep processes

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Tectonic processes in the lithosphere are plastic-elastic phenomena, in which the processes of plastic yield are in general inhomogeneous, and often strongly localized in faults, but then still, of course, at any one time inhomogeneous over the area of the fault. Motion on a fault may either be rare, sudden and seismic (stick-slip) or frequent and virtually aseismic (creep). Associated with these localized plastic strains, and dependent on their inhomogeneity, there are small elastic strains in the surrounding medium, in which sensitive strain gauges distributed over the surface, away from the fault, can give information about the inhomogeneities of plastic strain below the surface.

Present needs are above all to have Earth-strain gauges giving significant continuous records over a period of years, cheap enough and sufficiently undemanding in site requirement to permit installation in considerable numbers for the observation of one tectonic or pre-seismic phenomenon, and to learn their habits and how to separate tectonically significant signals from various kinds of noise, e.g. of hydrologic origin.

In planning such a layout, say, near to a strike-slip fault known to have creep activity, one sensibly assumes some simple model or models for the kind of motions on the fault which are to be looked for, and whose elastic consequences can be readily calculated. Single line dislocations moving on the fault provide such models, and enable one to see that features of the fault motion will be indicated by strain gauges in particular orientation. For example, if a travelling creep event is represented by vertical edge dislocation lines, strain gauges parallel to or perpendicular to the fault have their merits for resolving their distribution on the fault, but a strain gauge at 45° will see it coming from a greater distance (e.g. will see a slip-event of magnitude 1 mm when it is 10 km away, if a strain of 10^{-8} can be distinguished from noise), and will also respond to slip events which correspond to the rise of screw dislocations from below, which will not be noticed on strain gauges in the other two orientations.

Gradually accumulating screw dislocations at depth make a reasonable tentative mode for the stress build up preceding an eathquake.

The main content of my contribution derives from a discussion last September with Dr King and Dr Nason, about the desirable arrangement of a moderate number of strain gauges, 6 to 12 of them, to elucidate creep events on the Calaveras fault, a strike slip fault *en échelon* with the San Andreas fault, passing through the town of Hollister. One objective was to learn something about the motion in depth, and for this purpose one would certainly have strain gauges at various horizontal distances from the fault on the grounds that the deeper a dislocation is the wider would be the surface region in which its influence would be felt: but the question was what should be their orientation. There was quite a bit to be said for putting all or most of them in one orientation, so as to have only one variable, the distance from the fault, to bother about: Dr King suggested that orientation should be parallel to the fault. I was expressing a preference for an angle of 45° to the fault. Now let us look at the question a little closer.

Not really knowing what we have to look for, I suppose it sensible to postulate a simple model for the creep event, and plan a strain gauge lay-out which can confirm or deny that model, which can indicate in what manner we must complicate the model if the observations do not agree with it, and preferably one which will give some kind of response whatever the true model should be.

Perfectly adequate trial models for this purpose are to my mind simple line dislocations travelling on the fault. The slip vector of the dislocation is horizontal, directed along the strike of the fault. One very simple model then is a horizontal screw dislocation rising up from below.

The other extreme simple model is a vertical edge dislocation travelling along the fault. I will consider the second one first. The strain field for that dislocation, if it were in an infinite medium, is to be found in any text-book of dislocation theory: the strains are confined to the horizontal plane, and the strain in direction ϕ, at position (r, θ) relative to the dislocation, angles θ and ϕ both being measured from the slip direction, when the magnitude of the slip vector is b, is

$$e_\phi = \frac{-b}{4\pi(1-\nu)}\left\{(1-2\nu)\frac{\sin\theta}{r}-\frac{\cos\theta}{r}\sin 2(\phi-\theta)\right\}.$$

This is modified when we make our observations at the free surface, in a way calculated by Mrs Yoffe (1961), to

$$e_\phi = \frac{-b}{4\pi}\left\{(1-2\nu)\frac{\sin\theta}{r}-(1+2\nu)\frac{\cos\theta}{r}\sin 2(\phi-\theta)\right\}.$$

This is instructive. The plane dilatation, represented by the first term, is weakened by the factor $(1-\nu)$ and the shear strain which is the second term enhanced by the factor $(1-\nu)(1+2\nu)$. This is qualitatively the effect of the free surface in more general problems: area strains are weakened and shear strains enhanced and the changes are not drastic ones.

It is convenient to put this into cartesian coordinates: putting $r\sin\theta = y$, $r\cos\theta = x$, and $x/y = X$ we have

$$e_\phi = \frac{-b}{4\pi y}\left\{(1-2\nu)\frac{1}{1+X^2}+(1+2\nu)\left[\cos 2\phi\frac{2X^2}{(1+X^2)^2}+\sin 2\phi\frac{X(1-X^2)}{(1+X^2)^2}\right]\right\}$$
$$= \frac{-b}{4\pi y}\{(1-2\nu)F_1(X)+(1+2\nu)[\cos 2\phi F_2(X)+\sin 2\phi F_3(X)]\}.$$

Thus for any value of X, all strains are inversely proportional to y, the distance from the fault along which the dislocation travels. Of the three terms, the X-dependence of which is shown in figure 1, the first, which is independent of ϕ, is the smallest because of the weighting factor $(1-2\nu)$ compared with the factor $(1+2\nu)$ multiplying the other two. These are in the ratio 1 to 3 if we take the Poisson ratio ν as $\frac{1}{4}$. We can isolate either of the other two terms by choice, $F_2(X)$ made positive or negative at $\phi = 0$ or $90°$ and F_3, positive or negative at $\phi = 45°$ or $135°$.

At large X, F_1 and F_2 behave like X^{-2}: but F_3 has a longer range, behaving like X^{-1}.

$\phi = 0$ gives the simplest signal. For $\nu = \frac{1}{4}$ we have a strain of uniform sign, proportional to $\frac{1}{3}F_1+F_2$, a rather broad flat double-humped response as the dislocation goes by, without much character in the signal to distinguish it from noise. I suggest we take the slope, which measures rate of change of strain as the dislocation goes by, as a measure of character in the signal. On this criterion $\phi = 45°$ or $135°$, giving $\frac{1}{3}F_1+F_3$, wins with a slope of 1.111. $\phi = 90°$ giving $\frac{1}{3}F_1-F_2$, a symmetric signal with a slope of 1.023 (twice) comes next (it is perhaps surprising that this setting has any virtue), while $\phi = 0$ only scores a slope of 0.710. Nevertheless, these are not very large differences, and the signal from $\phi = 0$ despite its relative dullness does have the merit of uniform sign so that there will not be mutual cancellation from a distribution of dislocations of the same sign. Hence in the end I agreed with Dr King that he should have several strain gauges parallel to the fault at various distances from 1 to 15 km from it.

On the other hand, I stuck to my view that one or more strain gauges should be set at 45°. Besides giving a characteristic signal as the dislocation passes it gives the earliest warning that one is coming, because of its asymptotic X^{-1} behaviour. One such gauge, I argued, should be placed close to the fault (not *on* it, of course: there it will just get broken). For small y the strain signal is then $-(1+2\nu)\ b/4\pi x$. Thus if a strain of 10^{-8} is visible above the noise, as it surely should be, one should see a slip event of magnitude 1 mm coming when it is 15 km away: provided the dislocation extends to such a depth as 15 km, and if it does not show up till closer than that one has gained valuable depth information from a single instrument.

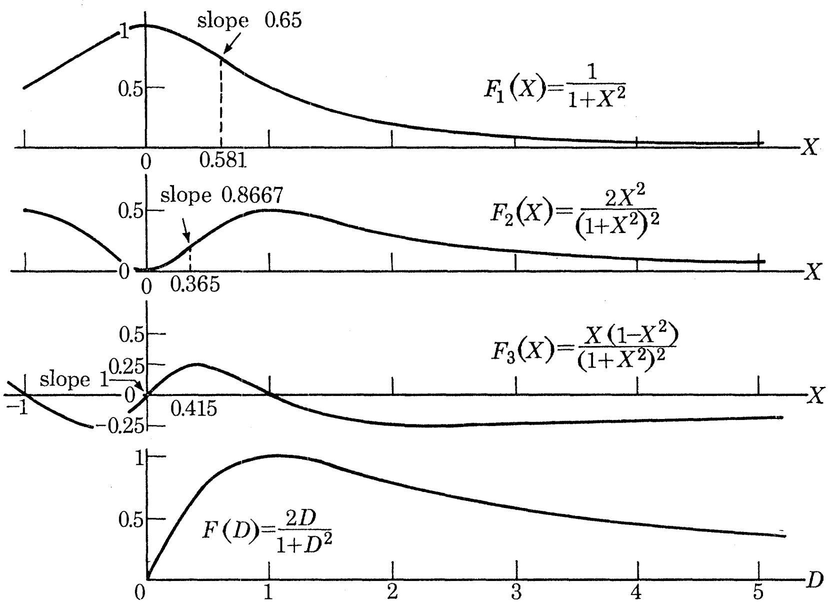

FIGURE 1

If the vertical edge dislocation extends to a finite depth d, it trails behind it a horizontal screw dislocation at that depth, and the strain remaining after it has gone by is that of the screw dislocation – a dilatationless shear:

$$e_{45^\circ} = e_{xy} = \frac{-b}{4\pi d}\,\frac{2}{1+Y^2},$$

where $Y = y/d$.

In this case the effect of the free surface is easily calculated by image theory and it is an enhancement of the shear by the factor 2 which appears in the numerator. The strain step, about 2×10^{-8} if $b = 1$ mm and $d = 10$ km, will not be seen at all with a strain gauge parallel or perpendicular to the fault.

If this is the proper model, one presumes that the screw dislocation subsequently migrates downwards, with a relaxation of its elastic strain as seen at surface. Alternatively, we should get just the same strain step if instead of trailing a screw dislocation behind it the travelling edge dislocation consumed a screw dislocation already present at depth d, which would presumably have risen from below with a gradual precursory strain. Or the whole phenomenon may be more nearly approximated by rising horizontal screw dislocations. These will escape detection unless we have strain gauges at 45°.

Mrs Yoffe's calculations in fact provide for straight dislocation lines intersecting the free surface at any angle, and this I suggest provides a sufficient range of possible models to use until we have more observations to constrain us to adopt more complicated models.

If I had appreciated sooner how much attention would be given to tilt measurement in this discussion, I would have given more attention to the changes of surface level attributable to any particular dislocation model. In the cases just discussed these arise as a Poisson ratio effect associated with the dilatational strains. The horizontal screw dislocation gives no change of surface level at all. The tilt therefore is one of the rather less informative quantities to measure, in so far as the Earth obeys simple elastic theory: it may, however, be very salutory to measure it wherever we are measuring strain components generally, to see how trustworthy simple elastic theory really is: Osborne Reynolds would tell us to anticipate swelling where the shear strains are. And also in this context it is worth remarking that assuming the truth of elasticity theory, the use of dislocation models for calculating changes of surface level may be unnecessarily cumbersome. We may get the answer we want by using a simple reciprocity theorem equivalent to Maxwell's reciprocal relations in thermodynamics.

If P_1 is load applied to a point on the Earth's surface and h_1 the depression there, τ_2 the component of shear stress on area A_2 of fault plane somewhere else, which acts in the direction of a relative displacement Δu_2 of the fault surfaces, then we can identify two terms in the change of internal energy of the Earth:

$$\mathrm{d}E = P_1\,\mathrm{d}h_1 + \tau_2\,\mathrm{d}(A_2\Delta u_2) + \ldots,$$

from which follows:

$$\frac{\mathrm{d}h_1}{\mathrm{d}(A_2\Delta u_2)} = -\frac{\mathrm{d}\tau_2}{\mathrm{d}P_1}.$$

The easiest way to calculate the change of land level in Greenland due to a given displacement on a fault in Alaska is to calculate, instead, the shear stress produced in Alaska due to a 1 kg weight deposited on Greenland.

Reference (Frank)

Yoffe, E. H. 1961 *Phil. Mag.* **6**, 1147.

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